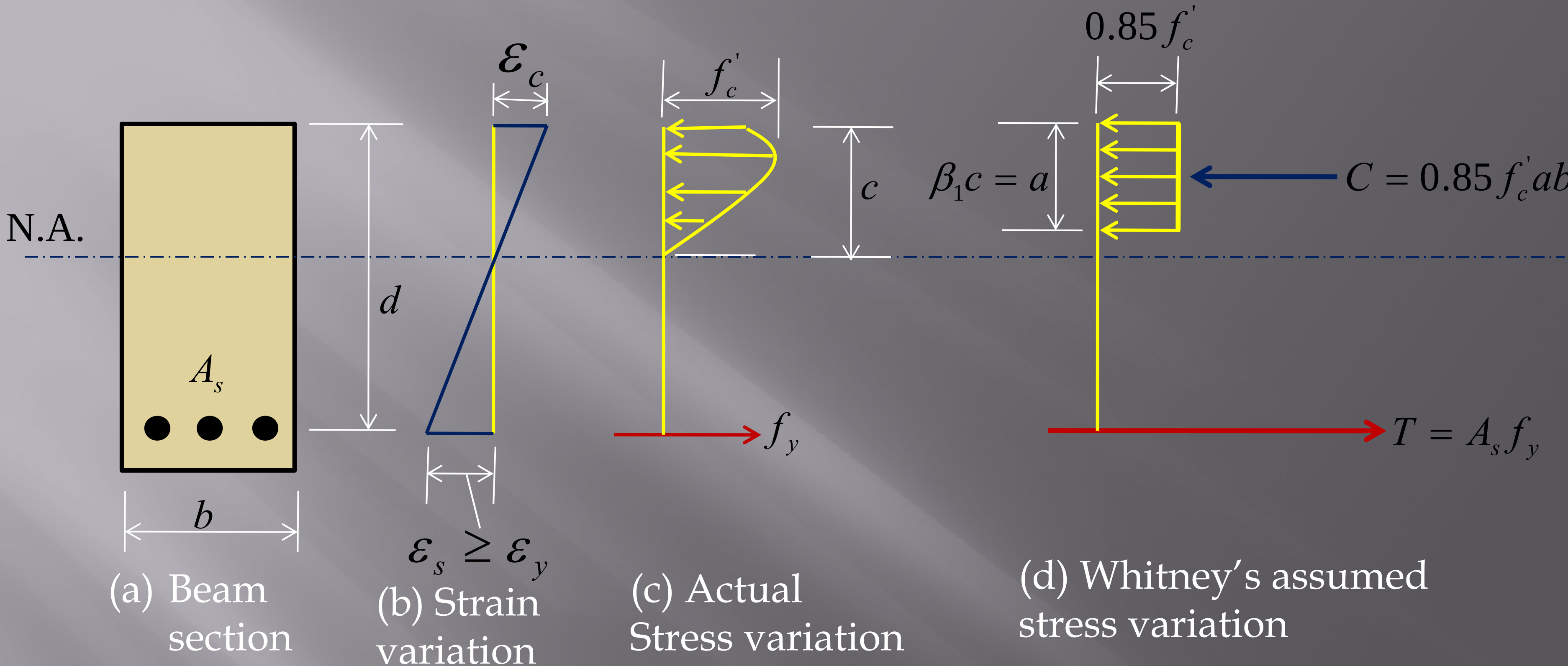
REINFORCED CONCRETE

(Design of Rectangular Beams)

(According to SBC/ACI Code)

Stress Distribution at Ultimate Load



Whitney replaced the curved stress block with an equivalent rectangular block of intensity $0.85 f_c'$ and depth $a = \beta_1 c$, where

For concretes with $f_c' \leq 30$ MPa, $\beta_1 = 0.85$; and

For concretes with $f_c' > 30$ MPa, $\beta_1 = 0.85 - 0.008(f_c' - 30) \geq 0.65$

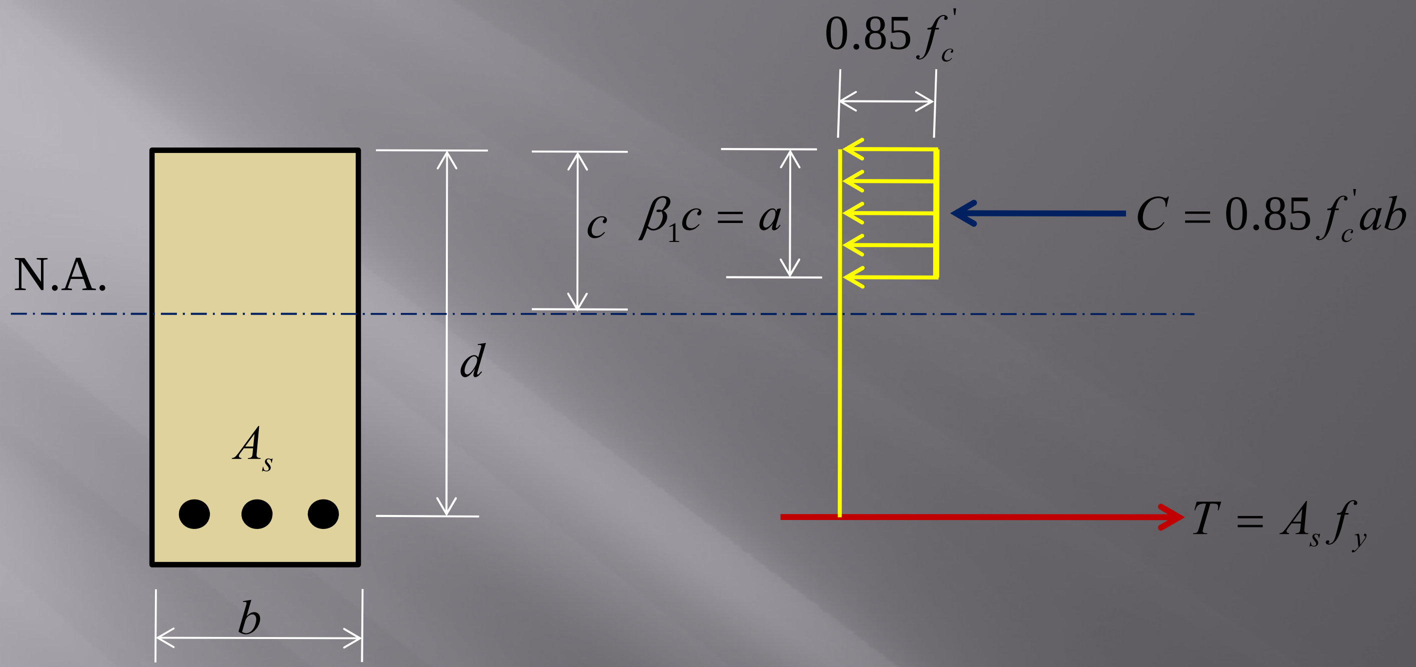
Area of the rectangular block = Area of the curved stress block.

The centroids of the two blocks coincide.

Note:

Concrete is assumed to crush at a strain of about 0.003 and the steel to yield at f_y.

Nominal and Design Flexural Strength



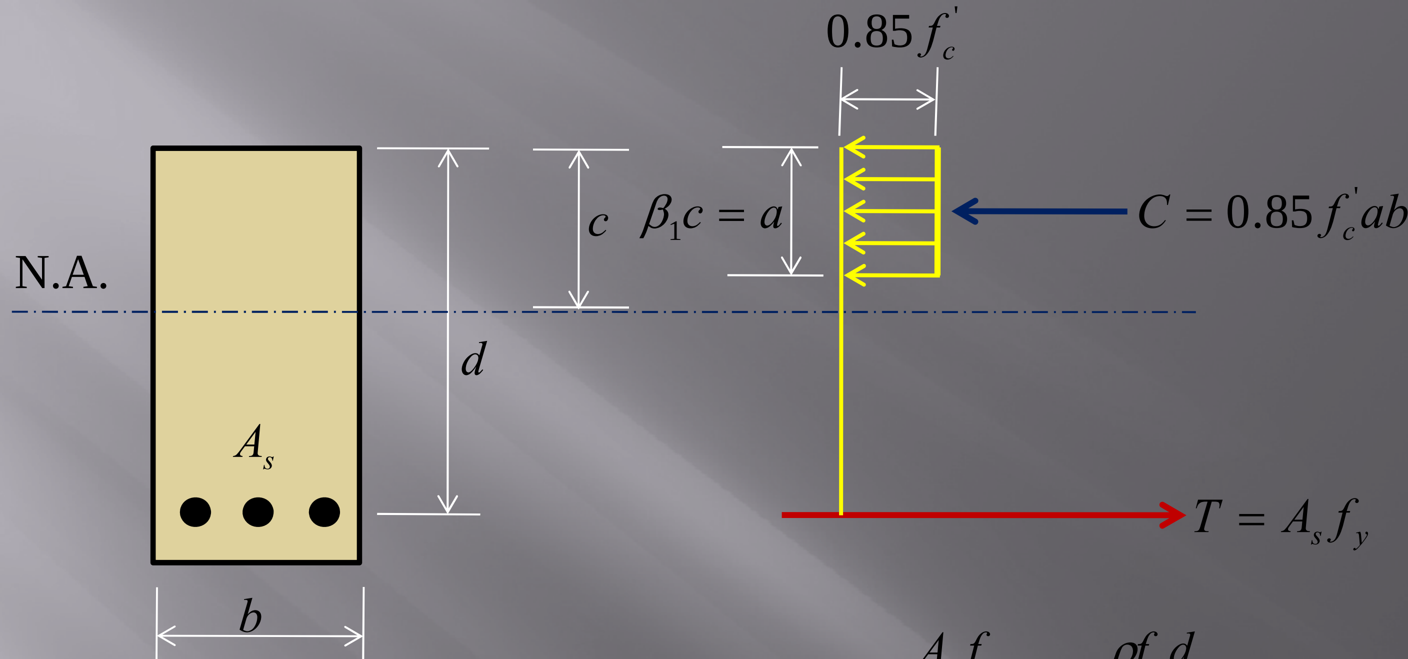
Theoretical or nominal flexural strength/resistance (or resisting moment) of a section is denoted by M_n.

The design/usable flexural strength/resistance =

Nominal flexural strength times the strength reduction factor i.e. ϕM_n.

The design flexural strength of a member, ϕM_n, must at least be equal to the calculated factored moment M_u, caused by the factored loads

$$\phi M_n \geq M_u$$

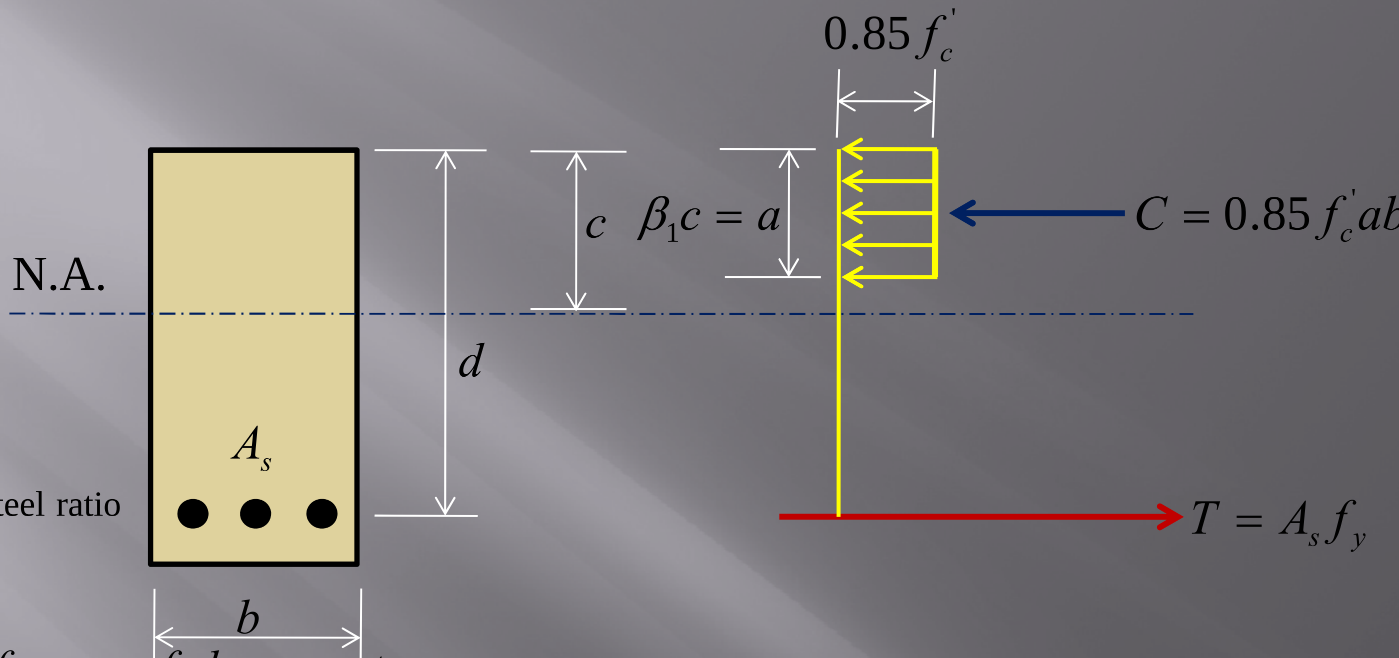


For equilibrium $C = T \Rightarrow 0.85 f_c' ab = A_s f_y \Rightarrow a = \dfrac{A_s f_y}{0.85 f_c' b} = \dfrac{\rho f_y d}{0.85 f_c'}$

where $\rho = \dfrac{A_s}{bd}$ = tensile steel ratio. Because the reinforcing steel is limited to an amount that it will yield well before the concrete reaches its ultimate strength.

Nominal moment : $M_n = T\left(d - \dfrac{a}{2}\right) \Rightarrow M_n = A_s f_y \left(d - \dfrac{a}{2}\right)$

Design strength : $\phi M_n = \phi A_s f_y \left(d - \dfrac{a}{2}\right)$



$\rho = \frac{A_s}{bd}$ = steel ratio

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{\rho f_y d}{0.85 f_c'}\,;\ \rho = \frac{A_s}{bd} \Rightarrow A_s = \rho bd\,;\ \phi M_n = M_u$$

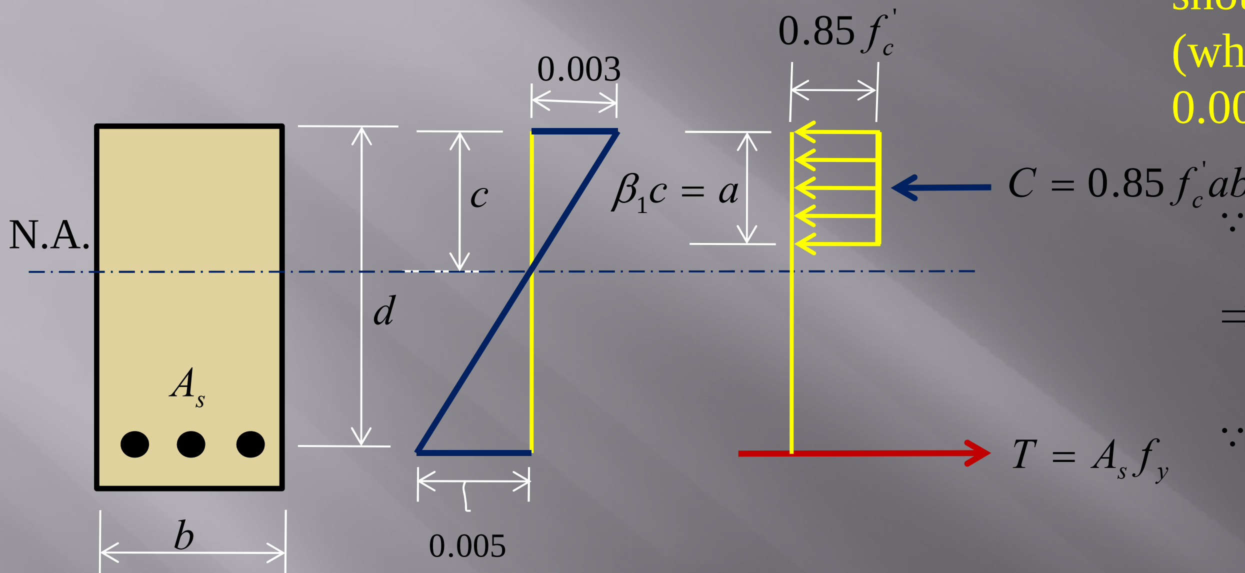
$$\text{Design strength}: \phi M_n = M_u = \phi A_s f_y \left(d - \frac{a}{2} \right) = \phi(\rho bd) f_y \left(d - \frac{\rho f_y d}{2 \times 0.85 f_c'} \right)$$

$$\Rightarrow M_u = \phi bd^2 f_y \rho \left(1 - \frac{\rho f_y}{1.7 f_c'} \right) \Rightarrow \frac{M_u}{\phi bd^2} = f_y \rho \left(1 - \frac{\rho f_y}{2 \times 0.85 f_c'} \right)$$

$$\Rightarrow \boxed{\rho = \frac{0.85 f_c'}{f_y} \left(1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}} \right) \quad \text{where } R_n = \frac{M_u}{\phi bd^2}}$$

Maximum Steel Ratio

In order to have the member ductile enough steel tensile strain should not be less than 0.005 (when the concrete strain reaches 0.003).



$$\because C = T \Rightarrow 0.85 f_c^{'} ab = A_s f_y$$

$$\Rightarrow a = \frac{A_s f_y}{0.85 f_c^{'} b} = \frac{\rho f_y d}{0.85 f_c^{'}} \text{ where } \rho = \frac{A_s}{bd};$$

$$\because a = c\beta_1 \Rightarrow c = \frac{a}{\beta_1} = \frac{\rho f_y d}{0.85 \beta_1 f_c^{'}}$$

$$\frac{c}{0.003} = \frac{d-c}{0.005} \Rightarrow \frac{c}{d} = \frac{0.003}{(0.003+0.005)}$$

$$\Rightarrow \frac{c}{d} = \frac{0.003}{(0.008)} \Rightarrow \frac{c}{d} = \frac{3}{8} \Rightarrow c = \frac{3}{8} d$$

$$\therefore \frac{\rho f_y d}{0.85 \beta_1 f_c^{'}} = \frac{3d}{8} \Rightarrow \rho = \left(\frac{0.85 \beta_1 f_c^{'}}{f_y}\right)\left(\frac{3}{8}\right)$$

$$\Rightarrow \boxed{\rho_{\max} = \left(\frac{0.85 \beta_1 f_c^{'}}{f_y}\right)\left(\frac{3}{8}\right)}$$

This is the maximum steel in order to have section fully ductile.

Minimum Percentage of Steel

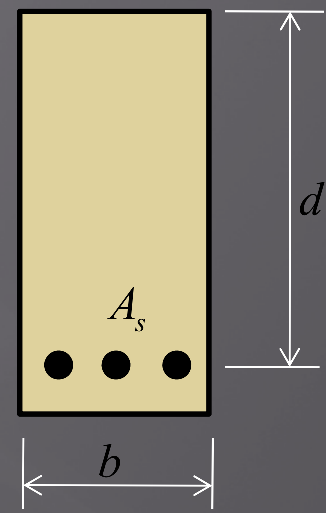
- Sometimes the applied bending moment (M_u) is so small that theoretically even a plane concrete section is able to resist it. However, If the ultimate resisting moment of the section is less than its cracking moment, the section will fail immediately when a crack occurs.
- This type of failure may occur without warning. To prevent such a possibility codes specify a certain amount of reinforcing that must be used at every section of flexural members.
- According to ACI (10.5.1):

$$A_{s,\min} = \left(\frac{\sqrt{f_c'}}{4 f_y} \right) b_w d \geq \left(\frac{1.4}{f_y} \right) b_w d$$

where b_w = web width of beams.

$$\because \rho_{\min} = \frac{A_{s,\min}}{b_w d} \Rightarrow \rho_{\min} = \left(\frac{\sqrt{f_c'}}{4 f_y} \right) \geq \left(\frac{1.4}{f_y} \right)$$

Steps in determining the design moment capacity



1a. Find stee ratio $\rho = \dfrac{A_s}{bd}$

1b. Find $\rho_{\min} = \left(\dfrac{\sqrt{f_c'}}{4f_y}\right) \geq \left(\dfrac{1.4}{f_y}\right)$

1c. Find $\rho_{\max} = \left(\dfrac{0.85\beta_1 f_c'}{f_y}\right)\left(\dfrac{3}{8}\right)$

1d. If $\rho_{\min} < \rho < \rho_{\max}$ OK. Go to the next step.

2. Find $a = \dfrac{A_s f_y}{0.85 f_c' b}$ and $c = \dfrac{a}{\beta_1}$

3a. Find strain in tensile steel $\varepsilon_t = \dfrac{d-c}{c}(0.003)$

3b. If $\varepsilon_t > 0.005$ Section is tension controlled $\Rightarrow \phi = 0.90$

If $0.002 < \varepsilon_t < 0.005$ Section is in transition zone

$$\Rightarrow \phi = 0.65 + (\varepsilon_t - 0.002)\left(\frac{250}{3}\right)$$

4. If section is tension controled or in transition zone

Design moment capacity $\phi M_n = \phi A_s f_y \left(d - \dfrac{a}{2}\right)$

Note: If $\varepsilon_t < 0.004$ Section is not ductile enough. Section is not suitable.

Steps to be followed in the design of a rectangular beam

1. Assume beam dimensions and estimate beam self weight

2. Compute factored load w_u and moment caused by the factored load i.e. M_u

3. Assume $\phi = 0.90$ and compute ρ (steel ratio) using

$$\rho = \frac{0.85 f_c'}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right) \quad \text{where } R_n = \frac{M_u}{\phi b d^2}$$

4. Find $As = \rho b d$ and select steel bars.

Check the solution :

i. Find $\rho = \frac{A_s}{bd}$; and check $\rho_{\min} < \rho < \rho_{\max}$

ii. Find $a = \frac{A_s f_y}{0.85 f_c'}$

iii. Find $\phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right)$ and check that $\phi M_n > M_u$

Example

Design a rectangular beam for a 6.5 m simple span if a dead load of 15kN/m (not including the beam weight) and a live load of 30 kN/m are to be supported. Use $f_c' = 30$ MPa and $f_y = 420$ MPa.

SOLUTION

Beam Dimensions :

Assume $h = l/10 = 6500/10 = 650$ mm

$\therefore d = h - \text{cover} = 650 - 50$ (use 40 mm in HW) $= 600$ mm

$b = 0.5h = 325$ mm (use 2/3 h in HW = 430 mm)

Self wt of the beam $= b \times h \times \gamma_{conc} = \frac{325}{1000} \times \frac{650}{1000} \times 25 = 5.28$ kN/m $(\gamma_{conc} = 25 \text{ kN/m}^3)$

(use in HW $\gamma_{conc} = 24 \text{ kN/m}^3$)

W_u and M_u : According to SBC 304 (use in HW $W_u = 1.4D + 1.7L$)

$(ACI)W_u = 1.2D + 1.6L = 1.2(15 + 5.28) + 1.6(30) = 72.336$ kN/m

$$M_u = \frac{\mathrm{W_u} l^2}{8} = \frac{72.336 \times 6.5^2}{8} = 382.0 \text{ kNm}$$

Assume $\phi = 0.90$

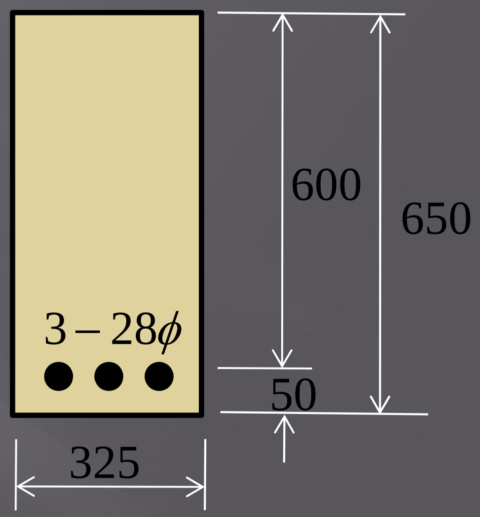
$$\rho = \frac{0.85 f_c'}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85 f_c'}}\right)$$

where $R_n = \dfrac{M_u}{\phi b d^2} = \dfrac{382.0 \times 10^6}{0.90 \times 325 \times 600^2} = 3.63$

$$\Rightarrow \rho = \frac{0.85 \times 30}{420}\left(1 - \sqrt{1 - \frac{2 \times 3.63}{0.85 \times 30}}\right) = 0.00937$$

$\therefore A_s = \rho b d = 0.00937 \times 325 \times 600 = 1827.2 \text{ mm}^2$

Use 28 mm bars, # of bars $= \dfrac{1827.2}{\frac{\pi}{4} \times 28^2} \approx 3 \Rightarrow$ Use 3 - 28ϕ bars.



Checking Solution

$$\text{Steel ratio } \rho = \frac{A_s}{bd} = \frac{3\times\left(\frac{\pi}{4}\times 28^2\right)}{325\times 600} = 0.00947$$

$$\rho_{\min} = \left(\frac{\sqrt{f_c'}}{4f_y}\right) \geq \left(\frac{1.4}{f_y}\right) \Rightarrow \rho_{\min} = \left(\frac{\sqrt{30}}{4\times 420}\right) \geq \left(\frac{1.4}{420}\right)$$

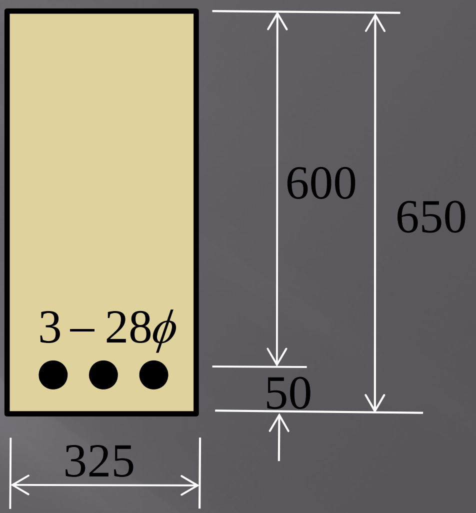
$$\Rightarrow \rho_{\min} = \left(\frac{\sqrt{30}}{4\times 420}\right) \geq \left(\frac{1.4}{420}\right) = 0.0033 \geq 0.0033$$

$$\Rightarrow \rho_{\min} = 0.0033$$

$$\rho_{\max} = \left(\frac{0.85\beta_1 f_c'}{f_y}\right)\left(\frac{3}{8}\right) \Rightarrow \rho_{\max} = \left(\frac{0.85\times 0.85\times 30}{420}\right)\left(\frac{3}{8}\right)$$

$$\Rightarrow \rho_{\max} = 0.0193$$

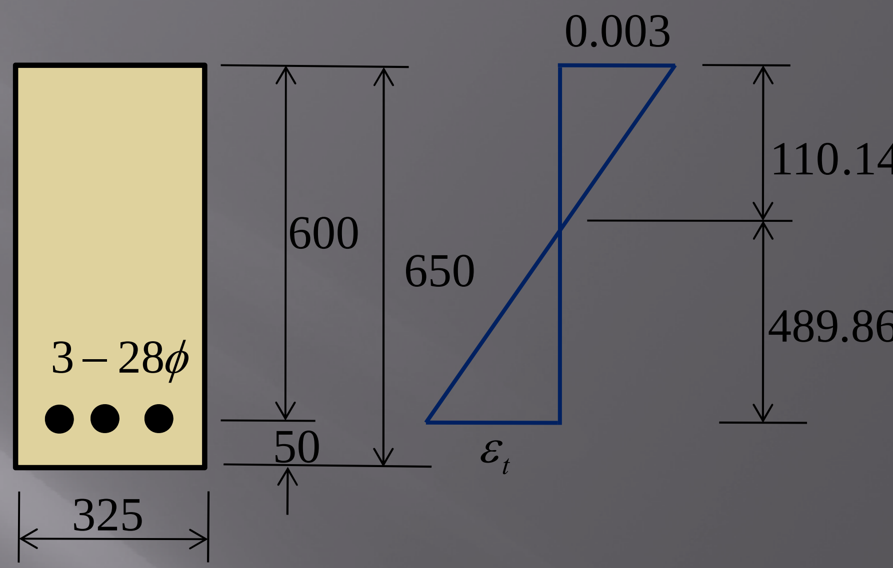
$\because \rho_{\min} < \rho < \rho_{\max}$ $\therefore$ Section is ductile and $\phi = 0.90$



Checking Solution Contd….

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3\times\left(\frac{\pi}{4}\times 28^2\right)\times 420}{0.85\times 30\times 325} = 93.62\text{ mm}$$

$$\because a = \beta_1 c \Rightarrow c = \frac{a}{\beta_1} = \frac{108.12}{0.85} = 110.14\text{ mm}$$



$$\varepsilon_t = \frac{d-c}{c}(0.003)$$

$$\Rightarrow \varepsilon_t = \frac{600-110.14}{110.14}(0.003) = 0.0133$$

$$\because \varepsilon_t = 0.0133 > 0.005$$

Section is tension controlled.

$\Rightarrow \phi = 0.90$ OK

As steel ratio is less than max. steel ratio. No need to check it again.

$$\therefore M_n = T\left(d - \frac{a}{2}\right) = A_s f_y\left(d - \frac{a}{2}\right)$$

$$\Rightarrow M_n = 1847.26\times 420\left(600 - \frac{93.62}{2}\right)$$

$$\Rightarrow M_n = 429.19\times 10^6\text{ Nmm} \Rightarrow M_n = 429.19\text{ kNm}$$

$$\therefore \text{Design moment capacity } \phi M_n = 0.9\times 429.19\text{ kNm}$$

$$\Rightarrow \phi M_n = 386.27\text{ kNm} > M_u (= 382.0\text{ kN.m})$$ OK

HW # 2

Design a rectangular beam for a 6.0 m simple span if a dead load of 15kN/m (not including the beam weight) and a live load of 30 kN/m are to be supported.

Use $f_c' = 30$ **MPa and** $f_y = 420$ **MPa.**

Use b = 430 mm

d = 610 mm

$\gamma_{conc} = 24\ kN/m^3$

40 mm concrete cover and Wu = 1.4Wd + 1.7Wl